

# Field-theoretic Methods in Strongly-Coupled Models of General Gauge Mediation

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An often-exploited feature of the operator product expansion (OPE) is that it incorporates a splitting of ultraviolet and infrared physics. In this paper we use this feature of the OPE to perform simple, approximate computations of soft masses in gauge-mediated supersymmetry breaking. The approximation amounts to truncating the OPEs for hidden-sector current-current operator products. Our method yields visible-sector superpartner spectra in terms of vacuum expectation values of a few hidden-sector IR elementary fields. We manage to obtain reasonable approximations to soft masses, even when the hidden sector is strongly coupled. We demonstrate our techniques in several examples, including a new framework where supersymmetry-breaking arises both from a hidden sector and dynamically.

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## 1. Introduction

The theoretical appeal of supersymmetry (SUSY) makes imperative the study of the phenomenology of its breaking. The Large Hadron Collider (LHC) has not yet found signs of low-scale SUSY, but abandoning SUSY at this early stage in experimental discovery would be premature. Nevertheless, SUSY extensions of the Standard Model are now tightly constrained by experimental data, and it appears that the simplest among them are not likely to survive as viable candidates for phenomenology. Therefore, new models of SUSY breaking as well as new tools for their analysis remain useful in exploring physics beyond the Standard Model. It would of course be ideal if tools were developed that could be used at strong coupling, since if SUSY is a symmetry of nature at some high scale, then it may very well reside in a model that is strongly-coupled at low energies.

In the context of gauge mediation of SUSY breaking (for a review see [1]) a formalism exists, known as general gauge mediation (GGM), that allows one to study such models in a unified fashion [2–4]. More specifically, SUSY-breaking parameters in the minimal supersymmetric standard model (MSSM) are generated in models of gauge-mediated SUSY breaking via two-point correlators of gauge-current superfields of the hidden, SUSY-breaking sector. This, then, dictates that a current analysis is possible, and allows one to understand the generation of soft masses in the MSSM Lagrangian.

Such an analysis benefits strongly from the use of the operator product expansion (OPE). In  $\mathcal{N} = 1$  superconformal theories OPEs of current correlators were studied in [5], where the superconformal symmetry was seen, as expected, to relate the OPEs of different components of the gauge-current superfield. Of course the study of the OPE is motivated by the fact that the OPE is one of the few tools that allows us to extract useful information even at strong coupling. This is reflected in the wealth of applications of the OPE in QCD.

The results of [5] were applied to the case of GGM correlators in [6]. Part of the motivation for that work was the observation that, even in theories that break the superconformal symmetry explicitly, one can introduce spurions to render the breaking spontaneous. The spurions are fully dynamical in the ultraviolet (UV), and an OPE analysis can be carried out to determine Wilson coefficients of spurionic operators in operator products. In the infrared (IR) the spurions acquire vacuum expectation values (vevs), and the Wilson coefficients have to be evolved from the UV according to their renormalization-group equation. It was shown in [6] that, in the case of minimal gauge mediation (MGM), soft masses could be approximated very well by only the leading spurionic term in the current-current OPE that develops a SUSY-breaking vev.

In MGM one can actually compute the full gaugino and sfermion masses using the OPE [6]. This is a rather special case and one cannot typically expect to be able to compute the complete current-current OPE. Nevertheless, it is physically acceptable to truncate the OPE and carry out the calculation of the soft masses, since the truncation is not expected to alter significantly

the essential results. The error introduced in truncating the OPE allows only an approximate determination of the soft masses, up to  $\mathcal{O}(1)$  overall factors which may be unimportant.

The technology developed in [6] may be used in strongly-coupled models of SUSY breaking. This is because the determination of Wilson coefficients is done in the UV, where asymptotic freedom allows for a perturbative computation, while non-perturbative effects are contained in the vevs of operators, i.e. are captured by IR quantities. Thus, at least at the qualitative level, one is able to use the methods of [6] in order to understand the generation of soft masses in the MSSM, even when the SUSY-breaking sector is strongly-coupled in the IR. In theories where weakly-coupled duals exist, it is also possible to check the strongly-coupled computations at the quantitative level by comparing results obtained with both methods. As we will see the approximations discussed here are indeed reasonable up to factors of order one, suggesting that relevant information can be extracted from them even in the strong-coupling regime.

Of course AdS/CFT [7] is another tool one can use in order to understand the behavior of field theories at strong coupling. Indeed, the realization of GGM in holography has been considered by numerous authors, see e.g. [8]. The main theme of these works is the description of GGM correlators by holographic methods. In this paper, however, our methods will be strictly field-theoretic and four-dimensional.

$\mathcal{N} = 1$  supersymmetric QCD (SQCD) is an ideal candidate for the application of our methods. In the free magnetic range of the massive theory Intriligator, Seiberg and Shih (ISS) demonstrated the existence of a metastable SUSY-breaking vacuum [9]. In their treatment they used the power of Seiberg duality [10] in order to establish their result in the strongly-coupled regime of the electric theory. The big global symmetry of SQCD in the ISS vacuum allows its use as the hidden SUSY breaking sector in the context of gauge mediation. Phenomenologically, however, there is a problem due to an accidental R-symmetry which precludes Majorana masses for the gauginos.

Although modifications of the ISS scenario have been proposed in the literature, see e.g. [11,12], in this paper we consider a new deformation where we add an additional spontaneous breaking of SUSY from a singlet chiral superfield. This superfield acquires its vev through its own dynamics, about which we will remain agnostic. This new model is similar to MGM but with messengers strongly interacting through another gauge group. As we will see, with this deformation our theory develops ISS-like vacua but with a broken R-symmetry. In our example there are no SUSY vacua anywhere in field space, but the ISS-like vacua we find should be metastable against decay to other SUSY-breaking vacua with lower energy.

The paper is organized as follows. In section 2 we review background material related to our work. We give a lightning review of  $\mathcal{N} = 1$  SQCD, as well as a quick overview of gauge mediation, GGM, and the role of the OPE in our considerations. In section 3 we present the analysis of our deformation of ISS. We also recover MGM and pure ISS as limits of our deformed SQCD.

We conclude in section 4. Appendix A contains weakly-coupled computations of the superpartner spectrum for general messenger sectors. We use notation and conventions of Wess & Bagger [13].

## 2. $\mathcal{N} = 1$ SQCD, gauge mediation of SUSY breaking, and the OPE

In this section we first review the aspects of  $\mathcal{N} = 1$  SQCD and gauge mediation which are necessary for our purposes. This section is far from self-contained and the reader is referred to the literature, e.g. [14], for completeness.

### 2.1. Essentials of $\mathcal{N} = 1$ SQCD

SQCD with  $N_c$  colors and  $N_f$  flavors is an  $\mathcal{N} = 1$  supersymmetric  $SU(N_c)$  gauge theory with  $N_f$  quark flavors  $Q^i$  (left-handed quarks) which are chiral superfields transforming in the  $\mathbf{N}_c$  of  $SU(N_c)$ , and  $N_f$  quark flavors  $\tilde{Q}_{\tilde{i}}$  (left-handed antiquarks) which are chiral superfields transforming in the  $\overline{\mathbf{N}}_c$  of  $SU(N_c)$ , where  $i, \tilde{i} = 1, \dots, N_f$  are flavor indices.<sup>1</sup>

There is a large global symmetry in SQCD—the relevant representations and charge assignments are shown in Table 1.

	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_A$	$U(1)_{R'}$
$Q$	$\mathbf{N}_f$	$\mathbf{1}$	1	1	1
$\tilde{Q}$	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	−1	1	1

**Table 1:** Matter content of SQCD and its (anomalous) transformation properties.

However, the  $U(1)_A \times U(1)_{R'}$  symmetry is anomalous. A single  $U(1)$  R-symmetry, which we will denote  $U(1)_R$ , survives and is a full quantum symmetry. Thus, the global symmetry of the quantum theory is  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$  with the appropriate R-charge assignment as shown in Table 2.

	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
$Q$	$\mathbf{N}_f$	$\mathbf{1}$	1	$1 - N_c/N_f$
$\tilde{Q}$	$\mathbf{1}$	$\overline{\mathbf{N}}_f$	−1	$1 - N_c/N_f$

**Table 2:** Matter content of SQCD and its (non-anomalous) transformation properties.

To make our notation more convenient we define the matrices

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<sup>1</sup>Note that there are no Fayet–Iliopoulos terms since the gauge group does not contain  $U(1)$  factors.

$$Q = \downarrow^a \left( \begin{pmatrix} \overset{i \longrightarrow}{\left( \begin{matrix} Q^1 \\ \vdots \\ Q^{N_f} \end{matrix} \right)} \end{pmatrix} \right), \quad \tilde{Q} = \downarrow^{\tilde{i}} \left( \begin{pmatrix} \overset{a \longrightarrow}{\left( \begin{matrix} \tilde{Q}^1 \\ \vdots \\ \tilde{Q}^{N_f} \end{matrix} \right)} \end{pmatrix} \right),$$

where  $a = 1, \dots, N_c$  is a fundamental or antifundamental color index. In this notation the Lagrangian of SQCD is<sup>2</sup>

$$\mathcal{L}_{\text{SQCD}} = \int d^4\theta \, \text{Tr}(Q^\dagger e^{2gV} Q + \tilde{Q} e^{-2gV} \tilde{Q}^\dagger) + \left( \int d^2\theta \, \text{tr} W^\alpha W_\alpha + \text{h.c.} \right).$$

In components (and after integrating out the auxiliary fields), this becomes

$$\begin{aligned} \mathcal{L}_{\text{SQCD}} = & -\text{tr}(\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda\sigma^\mu\mathcal{D}_\mu\bar{\lambda}) - \text{Tr}[\mathcal{D}_\mu Q^\dagger \mathcal{D}^\mu Q + \mathcal{D}_\mu \tilde{Q} \mathcal{D}^\mu \tilde{Q}^\dagger + i\bar{\psi}\bar{\sigma}^\mu\mathcal{D}_\mu\psi + i\tilde{\psi}\tilde{\sigma}^\mu\mathcal{D}_\mu\tilde{\psi} \\ & - i\sqrt{2}g(Q^\dagger\lambda\psi - \bar{\psi}\bar{\lambda}Q - \tilde{\psi}\lambda\tilde{Q}^\dagger + \tilde{Q}\bar{\lambda}\tilde{\psi})] - \tfrac{1}{2}g^2 \sum_{I=1}^{N_c^2-1} [\text{Tr}(Q^\dagger T^I Q - \tilde{Q} T^I \tilde{Q}^\dagger)]^2, \end{aligned}$$

where  $\mathcal{D}_\mu = \partial_\mu + igA_\mu^I T^I(R)$  is the gauge-covariant derivative. Note that SQCD only has D-term contributions to the scalar potential,

$$\mathcal{V}_{\text{SQCD}} = \tfrac{1}{2}g^2 \sum_{I=1}^{N_c^2-1} [\text{Tr}(Q^\dagger T^I Q - \tilde{Q} T^I \tilde{Q}^\dagger)]^2,$$

where  $T^I$  are  $SU(N_c)$  generators with  $I = 1, \dots, N_c^2 - 1$  the adjoint color index. This scalar potential has a large vacuum degeneracy, which is however lifted when masses for the quarks are added.

### 2.1.1. Masses for the flavors

The lowest-dimensional gauge-invariant chiral superfield one can construct from  $Q^i$  and  $\tilde{Q}_{\tilde{i}}$ , namely the mesonic superfield<sup>3</sup>

$$M_i^{\tilde{i}} = \text{Tr}(\tilde{Q}_{\tilde{i}} Q^i)_{(N_c, 0)},$$

can be used to give gauge-invariant masses to all quark flavors. The Lagrangian of massive SQCD (mSQCD) is then

$$\mathcal{L}_{\text{mSQCD}} = \mathcal{L}_{\text{SQCD}} + \left( \int d^2\theta \, W_{\text{tree}} + \text{h.c.} \right),$$

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<sup>2</sup>Tr denotes a sum over both fundamental gauge and flavor indices, while tr denotes a sum over adjoint gauge indices only, e.g.

$$\text{Tr} Q^\dagger T^I Q \equiv Q_{ib}^\dagger (T^I)_c^b Q^{ic} \quad \text{and} \quad \text{tr} W^\alpha W_\alpha \equiv W^{\alpha I} W_\alpha^I.$$

<sup>3</sup>Tr $(\cdot)_{(x,y)}$  denotes a sum over color indices up to  $x$  and flavor indices up to  $y$ . Hence,  $\text{Tr}(\cdot) \equiv \text{Tr}(\cdot)_{(N_c, N_f)}$ .

where  $W_{\text{tree}} = \text{Tr}(mM)_{(0,N_f)}$ , with  $m$  a nondegenerate  $N_f \times N_f$  mass matrix. Note that the inclusion of masses breaks the non-Abelian part of the global symmetry to one of its subgroups. The scalar potential in  $\mathcal{L}_{\text{mSCQD}}$  is

$$\mathcal{V}_{\text{mSCQD}} = \text{Tr}(mm^\dagger Q^\dagger Q + m^\dagger m \tilde{Q} \tilde{Q}^\dagger) + \frac{1}{2}g^2 \sum_{I=1}^{N_c^2-1} [\text{Tr}(Q^\dagger T^I Q - \tilde{Q} T^I \tilde{Q}^\dagger)]^2,$$

and includes the anticipated mass terms. The vacuum degeneracy of  $\mathcal{V}_{\text{SCQD}}$  is lifted in  $\mathcal{V}_{\text{mSCQD}}$  due to the mass terms.

## 2.2. Essentials of gauge mediation

Mediation of SUSY breaking was born to address phenomenological impasses reached by trying to break SUSY within the observable sector of supersymmetric extensions of the standard model. As an example, supertrace conditions that remain even after SUSY is broken are hard to satisfy consistently with the observed low-mass spectrum of particles [15].

Gauge mediation requires that SUSY be broken in a hidden sector with the breaking communicated to the MSSM through the familiar gauge interactions, thus avoiding new sources of flavor-changing neutral currents, a generic problem in models of gravity-mediated SUSY breaking. All soft SUSY-breaking terms in the MSSM Lagrangian are generated via loop effects, and desired phenomenology is obtained very naturally, except, of course, for the notorious  $\mu/B_\mu$  problem [16]. For an extensive review of theories with gauge mediation the reader is referred to [1].

In the minimal incarnation of gauge mediation one assumes the existence of a hidden sector that contains a gauge singlet chiral superfield  $S$ , as well as a messenger sector with fields  $\Phi, \tilde{\Phi}$  in complete GUT representations so that gauge-coupling unification is not spoiled. Through interactions in the hidden sector  $S$  develops a vev both in its first and its last component,  $\langle S \rangle = \langle S \rangle + \theta^2 \langle F_S \rangle$ . The superpotential that couples the hidden sector with the messenger sector is  $W_{\text{h}\otimes\text{m}} \propto S \text{Tr}(\tilde{\Phi}\Phi)$ , such that the SUSY breaking of the hidden sector is fed into the messenger sector. The usual gauge interactions then communicate the SUSY breaking to the supersymmetric extension of the standard model generating the appropriate soft SUSY-breaking terms.

### 2.2.1. General gauge mediation

A unified and powerful framework for the study of gauge mediation, dubbed general gauge mediation, was developed in [2–4]. In GGM soft terms are written in terms of one- and two-point correlators of components of a current (linear) superfield of the hidden sector,

$$\mathcal{J}(z) = J(x) + i\theta j(x) - i\bar{\theta}\bar{j}(x) - \theta\sigma^\mu\bar{\theta}j_\mu(x) + \cdots, \quad (2.1)$$

where the ellipsis stands for derivative terms, following from the conservation equations  $\mathcal{D}^2 \mathcal{J} = \bar{\mathcal{D}}^2 \mathcal{J} = 0$ .<sup>4</sup> Among the virtues of GGM is its ability to disentangle genuine characteristics of gauge mediation from possible model-dependent features. GGM also leads to phenomenological superpartner-mass sum rules that, if verified by the LHC, will identify gauge mediation as the dominant means by which SUSY is broken in nature (see e.g. [17] for a renormalization group study of the above-mentioned sum rules). Moreover, GGM encompasses strongly-coupled hidden sectors at the qualitative level and also at the quantitative level, at least in principle. In our view this is the greatest strength of GGM, which is nevertheless largely unexplored. In the next section it will be discussed extensively.

The correlators one considers in GGM are (using the conventions of [4])

$$\begin{aligned}\langle J(x)J(0) \rangle &= C_0(x) \xrightarrow{\text{F.T.}} \tilde{C}_0(p), \\ \langle j_\alpha(x)\bar{j}_{\dot{\alpha}}(0) \rangle &= -i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu C_{1/2}(x) \xrightarrow{\text{F.T.}} \sigma_{\alpha\dot{\alpha}}^\mu p_\mu \tilde{C}_{1/2}(p), \\ \langle j_\mu(x)j_\nu(0) \rangle &= (\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)C_1(x) \xrightarrow{\text{F.T.}} -(\eta_{\mu\nu}p^2 - p_\mu p_\nu)\tilde{C}_1(p), \\ \langle j_\alpha(x)j_\beta(0) \rangle &= \epsilon_{\alpha\beta}B_{1/2}(x) \xrightarrow{\text{F.T.}} \epsilon_{\alpha\beta}\tilde{B}_{1/2}(p),\end{aligned}\tag{2.2}$$

where F.T. stands for Fourier-transforming,  $\text{F.T.} \equiv i \int d^4x e^{-ip \cdot x}$ . It was realized in [3] that for the soft masses, for example, only the one-point function  $\langle J(x) \rangle$  and the correlator  $\langle J(x)J(0) \rangle$  are needed:<sup>5</sup>

$$\begin{aligned}M_{\text{gaugino}} &= \frac{i\pi\alpha_{\text{SM}}}{d(G)} \int d^4x \langle \mathcal{Q}^2(J^A(x)J^A(0)) \rangle, \\ m_{\text{fermion}}^2 &= 4\pi Y \alpha_{\text{SM}} \langle J(x) \rangle + \frac{iC_2(R)\alpha_{\text{SM}}^2}{8d(G)} \int d^4x \ln(x^2 M_{\text{m}}^2) \langle \bar{\mathcal{Q}}^2 \mathcal{Q}^2(J^A(x)J^A(0)) \rangle,\end{aligned}\tag{2.3}$$

where  $M_{\text{m}}$  is a supersymmetric scale in the hidden-sector theory, e.g. the messenger scale. For clarity, the appropriate MSSM gauge group index  $A$  has been reintroduced.<sup>6</sup>

Using the results of [18] it was pointed out in [5] that, within a superconformal field theory, the superconformal algebra and current conservation are powerful enough to relate all possible two-operator products of components of the current superfield (2.1) to the operator product  $J(x)J(0)$ . Consequently, only the correlator  $\langle J(x)J(0) \rangle$  is necessary, while all other correlators in (2.2) can be expressed in terms of  $\langle J(x)J(0) \rangle$  with the help of the superconformal group. From [5] one has

$$j_\alpha(x)j_\beta(0) = \frac{1}{x^2} \mathcal{Q}_\beta(ix \cdot \sigma \bar{\mathcal{S}})_\alpha(J(x)J(0)),$$

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<sup>4</sup>In this paper  $D$  is the D-term, thus we use  $\mathcal{D}$  for the covariant derivatives.

<sup>5</sup>Since  $Q$  is used in this paper for the quarks of SQCD, we use  $\mathcal{Q}$  to denote the SUSY generator.  $\mathcal{Q}$  always acts with an adjoint action, e.g.  $\mathcal{Q}^2(\mathcal{O}(x)) \equiv \{\mathcal{Q}^\alpha, [\mathcal{Q}_\alpha, \mathcal{O}(x)]\}$ .

<sup>6</sup>The MSSM gauge group is chosen to be a GUT  $SU(N)$  subgroup of the hidden-sector global symmetry group where  $A = 1, \dots, N^2 - 1$  is the appropriate adjoint index.

$$\begin{aligned}
j_\alpha(x)\bar{j}_{\dot{\alpha}}(0) &= \frac{1}{x^4} \left[ (\mathcal{S} ix \cdot \sigma)_{\dot{\alpha}} (ix \cdot \sigma \bar{\mathcal{S}})_{\alpha} - x^2 \bar{\mathcal{Q}}_{\dot{\alpha}} (ix \cdot \sigma \bar{\mathcal{S}})_{\alpha} + 2\Delta_J x^2 (ix \cdot \sigma)_{\alpha\dot{\alpha}} \right] (J(x)J(0)), \\
j_\mu(x)j_\nu(0) &= \frac{1}{16x^8} \left[ (x^2\eta_{\mu\rho} - 2x_\mu x_\rho)(\mathcal{S}\sigma^\rho \bar{\mathcal{S}} - \bar{\mathcal{S}}\sigma^\rho \mathcal{S}) \right. \\
&\quad \times \{x^4(\bar{\mathcal{Q}}\bar{\sigma}_\nu \mathcal{Q} - \mathcal{Q}\sigma_\nu \bar{\mathcal{Q}}) + (x^2\eta_{\nu\lambda} - 2x_\nu x_\lambda)(\mathcal{S}\sigma^\lambda \bar{\mathcal{S}} - \bar{\mathcal{S}}\sigma^\lambda \mathcal{S}) \\
&\quad \left. - 2x^2(\mathcal{Q}\sigma_\nu ix \cdot \bar{\sigma} \mathcal{S} - \bar{\mathcal{Q}}\bar{\sigma}_\nu ix \cdot \sigma \bar{\mathcal{S}}) \right] \\
&\quad - 8i(\Delta_J + 1)x^2(\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda} - i\epsilon_{\mu\nu\lambda\rho})x^\lambda \\
&\quad \times \{(x^2\eta^{\rho\delta} - 2x^\rho x^\delta)\mathcal{S}\sigma_\delta \bar{\mathcal{S}} + x^2\bar{\mathcal{Q}}\bar{\sigma}^\rho ix \cdot \sigma \bar{\mathcal{S}} + 4i\Delta_J x^2 x^\rho\} \\
&\quad - 8i(\Delta_J + 1)x^2(\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda} + i\epsilon_{\mu\nu\lambda\rho})x^\lambda \\
&\quad \times \{(x^2\eta^{\rho\delta} - 2x^\rho x^\delta)\bar{\mathcal{S}}\bar{\sigma}_\delta \mathcal{S} + x^2\mathcal{Q}\sigma^\rho ix \cdot \bar{\sigma} \mathcal{S} + 4i\Delta_J x^2 x^\rho\} \\
&\quad \left. + 32x^4\Delta_J(\Delta_J + 1)(x^2\eta_{\mu\nu} - 2x_\mu x_\nu) \right] (J(x)J(0)),
\end{aligned}$$

with  $\mathcal{S}, \bar{\mathcal{S}}$  the superconformal supercharges. Implications of this observation in the case of a UV asymptotically-free hidden sector (i.e. with approximate superconformal symmetry) and in particular in the example of MGM were analyzed using the OPE in [6], and we will rely heavily here on the results of that paper. It is important to note that using the OPE in the equations above and Fourier-transforming the results allow a simple evaluation of the total cross-sections of the visible sector to the hidden sector, with different mediators corresponding to the different components of the MSSM vector superfields. This is reminiscent of electron-positron scattering to hadrons in QCD. In the following we will focus on the superpartner spectrum, and will not discuss such cross-sections.

As shown in [6] a complete expansion of (2.3) can be obtained with the help of the  $J(x)J(0)$  OPE which thus gives an approximation to the soft MSSM SUSY-breaking masses even for strongly-coupled hidden sectors. The expansion relies on several approximations (e.g. cuts at supersymmetric threshold, uniform convergence of the OPE) but, at least in the simple case of MGM, a complete knowledge of the OPE leads to an exact evaluation of the soft SUSY-breaking masses, after analytic continuation of the sums. To avoid complications such as arduous OPE computations and analytic continuations, a further approximation to (2.3), given by

$$\begin{aligned}
M_{\text{gaugino}} &\approx -\frac{\pi w^{AA}\alpha_{\text{SM}}}{8d(G)M_{\text{m}}^2}\gamma_{Ki}\langle\mathcal{Q}^2(\mathcal{O}_i(0))\rangle, \\
m_{\text{sfermion}}^2 &\approx 4\pi Y\alpha_{\text{SM}}\langle J(x)\rangle + \frac{C_2(R)w^{AA}\alpha_{\text{SM}}^2}{64d(G)M_{\text{m}}^2}\gamma_{Ki}\langle\bar{\mathcal{Q}}^2\mathcal{Q}^2(\mathcal{O}_i(0))\rangle,
\end{aligned} \tag{2.4}$$

was introduced in [6]. Here  $w^{AB}$  is the OPE coefficient of a scalar operator  $K$  with classical scaling dimension 2 in the OPE of two conserved currents (like, e.g. the Konishi current in MGM), and  $\gamma$  is the anomalous-dimension matrix of  $K$  (see (3.2), (3.3) and (3.7)). So, to get an approximation to the soft MSSM SUSY-breaking masses, *even in a theory with a strongly-coupled*



*hidden sector*, one only needs to identify the lowest-dimension operators that have non-zero vevs after acted upon with  $\mathcal{Q}^2$  and  $\bar{\mathcal{Q}}^2\mathcal{Q}^2$ .

In the example of MGM there is only one such operator, namely  $S^\dagger S$ , and calculating its mixing with the Konishi current one finds that the approximation to the soft masses (2.4) is actually only a factor of 2 smaller than the usually quoted answers [19]. For more details the reader is referred to section 3.1 and [6].

### 3. SQCD as the SUSY-breaking sector

To be specific, in this paper we take the messenger sector of gauge mediation to be SQCD without masses for the quarks but, instead, with Kähler potential and superpotential for matter fields given by

$$\begin{aligned} K_e &= \text{Tr}(Q^\dagger Q + \tilde{Q}\tilde{Q}^\dagger) + S^\dagger S, \\ W_e &= \xi S \text{Tr} \tilde{Q} Q, \end{aligned} \tag{3.1}$$

where  $S$  is the MGM-like singlet field which has non-vanishing vacuum expectation value  $\langle S \rangle = \langle S \rangle + \theta^2 \langle F_S \rangle$ , and  $Q, \tilde{Q}$  are the messenger fields which are  $N_f$  flavors of  $SU(N_c)$  fundamental and antifundamental superfields. The non-Abelian part of the global symmetry of SQCD is thus broken to its diagonal subgroup,  $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ , which contains  $SU(N)$ , a grand-unified extension of the MSSM gauge group. The coupling  $\xi$  is assumed weak. We will refer to SQCD with an extra singlet and the superpotential (3.1) as sSQCD. We stress that it is straightforward to repeat the analysis for more general messenger sectors and hidden sectors.

In order to use the approximation (2.4) in this framework, it is necessary to determine the  $J(x)J(0)$  OPE at the lowest non-trivial order as well as the appropriate anomalous dimension matrix.

Note that non-perturbative effects (instantons) contribute both to the vevs of operators appearing on the right-hand side of the OPE and to the (perturbative) OPE coefficients themselves [20]. Furthermore, for operator products satisfying the chirality selection rule, instantons can lead to new non-perturbative contributions on the right-hand side of the OPE, i.e. operators with purely non-perturbative OPE coefficients [21]. Instanton corrections of the first type do not modify the OPE coefficients at lowest order and are thus non-negligible only for vevs of operators. Instanton corrections of the second type lead to new non-perturbative OPE contributions which can dominate over the perturbative ones.<sup>7</sup> Since the  $J(x)J(0)$  OPE is non-trivial at the

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<sup>7</sup>It is important to notice that both types of non-perturbative contributions to the OPE coefficients are calculable. Thus, as usual, the OPE coefficients are fully calculable, while all incalculable non-perturbative effects are contained in the vevs of operators.

classical level and does not satisfy the chirality selection rule, for our purposes non-perturbative contributions that are calculable can be safely ignored.

The currents of interest for the evaluation of the  $J(x)J(0)$  OPE are

$$J^A = \text{Tr}(Qt^A Q^\dagger - \tilde{Q}^\dagger t^A \tilde{Q})_{(N_c, N)},$$

$$K = \text{Tr}(Q^\dagger Q + \tilde{Q} \tilde{Q}^\dagger)_{(N_c, N)},$$

where we denote the  $SU(N)$  generators by  $t^A$  to avoid confusion with the  $SU(N_c)$  generators  $T^I$ . At the classical level the OPE is simply

$$J^A(x)J^B(0) = \frac{N_c \delta^{AB}}{16\pi^4 x^4} \mathbb{1} + \frac{w^{AB}}{4\pi^2 x^2} K(0) + \dots, \quad (3.2)$$

where  $w^{AB} = \delta^{AB}/N$ , while the one-loop anomalous-dimension matrix between  $K$  and  $S^\dagger S$  is

$$\gamma = \begin{pmatrix} \gamma_{K,K} & \gamma_{K,S^\dagger S} \\ \gamma_{S^\dagger S,K} & \gamma_{S^\dagger S,S^\dagger S} \end{pmatrix} \xrightarrow[\text{coupling}]{\text{weak}} \frac{1}{8\pi^2} \begin{pmatrix} 2C_2(N_c)g^2 & 2NN_c|\xi|^2 \\ |\xi|^2 & 0 \end{pmatrix}. \quad (3.3)$$

Note here that although computable in the weak-coupling regime, the anomalous dimensions are large in the IR for strongly-coupled theories and are therefore kept undetermined in the following. The soft SUSY-breaking masses are

$$M_{\text{gaugino}} \approx -\frac{\pi\alpha_{\text{SM}}}{8N|\xi\langle S \rangle|^2} \left[ \gamma_{K,K} \langle \mathcal{Q}^2(K) \rangle + \gamma_{K,S^\dagger S} \langle \mathcal{Q}^2(S^\dagger S) \rangle \right],$$

$$m_{\text{sfermion}}^2 \approx \frac{C_2(R)\alpha_{\text{SM}}^2}{64N|\xi\langle S \rangle|^2} \left[ \gamma_{K,K} \langle \bar{\mathcal{Q}}^2 \mathcal{Q}^2(K) \rangle + \gamma_{K,S^\dagger S} \langle \bar{\mathcal{Q}}^2 \mathcal{Q}^2(S^\dagger S) \rangle \right], \quad (3.4)$$

since the supersymmetric mass scale  $M_m = |\xi\langle S \rangle|$  and  $\langle J \rangle = 0$  for a non-Abelian group.

These expressions can be further simplified using the supersymmetry algebra and the Konishi anomaly [22] (in Wess–Zumino gauge) in the  $\alpha_{\text{SM}} \rightarrow 0$  limit:

$$\mathcal{Q}^2(S^\dagger S) = 4S^\dagger F_S,$$

$$\bar{\mathcal{Q}}^2 \mathcal{Q}^2(S^\dagger S) = 16(F_S^\dagger F_S - i\bar{\psi}_S \bar{\sigma}^\mu \partial_\mu \psi_S + S^\dagger \partial^2 S),$$

$$\mathcal{Q}^2(K) = 4 \left[ \text{Tr}(Q^\dagger F + \tilde{F} \tilde{Q}^\dagger)_{(N_c, N)} + \frac{Ng^2}{16\pi^2} \text{tr} \bar{\lambda} \lambda \right],$$

$$\bar{\mathcal{Q}}^2 \mathcal{Q}^2(K) = 16 \left[ \text{Tr}(F^\dagger F - i\bar{\psi} \bar{\sigma}^\mu \mathcal{D}_\mu \psi + Q^\dagger \mathcal{D}^2 Q + i\sqrt{2}g(Q^\dagger \lambda \psi - \bar{\psi} \bar{\lambda} Q) + gQ^\dagger DQ)_{(N_c, N)} \right. \\ \left. + \{(Q, \psi, F, g) \rightarrow (\tilde{Q}, \tilde{\psi}, \tilde{F}, -g)\} - \frac{Ng^2}{32\pi^2} \text{tr}(2DD - 4i\lambda\sigma^\mu \mathcal{D}_\mu \bar{\lambda} - F_{\mu\nu} F^{\mu\nu}) \right].$$

Note that  $\bar{\mathcal{Q}}^2 \mathcal{Q}^2(S^\dagger S, K)$  are real up to total derivatives. After using the equations of motion (we omit the ones for the fields with a tilde),

$$F = -\xi^* S^\dagger \tilde{Q}^\dagger, \quad D^I = -g \text{Tr}(Q^\dagger T^I Q - \tilde{Q} T^I \tilde{Q}^\dagger),$$

$$\mathcal{D}^2 Q = -i\sqrt{2}g\lambda\psi - gDQ + |\xi|^2 S^\dagger SQ - \xi^* F_S^\dagger \tilde{Q}^\dagger,$$

$$i\bar{\sigma}^\mu \mathcal{D}_\mu \psi = -i\sqrt{2}g\bar{\lambda}Q - \xi^* S^\dagger \bar{\tilde{\psi}}, \quad i\sigma^\mu \mathcal{D}_\mu \bar{\lambda}^I = i\sqrt{2}g(Q^\dagger T^I \psi - \tilde{\psi} T^I \tilde{Q}^\dagger),$$

the approximations (3.4) can be written in terms of vacuum condensates of UV elementary fields as

$$\begin{aligned}
M_{\text{gaugino}} &\approx \frac{\pi\alpha_{\text{SM}}}{2N|\xi\langle S \rangle|^2} \left[ 2\xi^* \gamma_{K,K} \langle S^\dagger \text{Tr}(Q^\dagger \tilde{Q}^\dagger)_{(N_c, N)} \rangle - \frac{Ng^2}{16\pi^2} \gamma_{K,K} \langle \text{tr } \bar{\lambda} \lambda \rangle - \gamma_{K, S^\dagger S} \langle S^\dagger F_S \rangle \right] \\
m_{\text{sfermion}}^2 &\approx \frac{C_2(R)\alpha_{\text{SM}}^2}{4N|\xi\langle S \rangle|^2} \left[ 2\xi^* \gamma_{K,K} \langle \xi S^\dagger S K + \text{Tr}(S^\dagger \bar{\psi} \bar{\psi} - F_S^\dagger Q^\dagger \tilde{Q}^\dagger)_{(N_c, N)} \rangle \right. \\
&\quad \left. - \frac{Ng^2}{32\pi^2} \gamma_{K,K} \langle \text{tr}(2DD - 4E - F_{\mu\nu} F^{\mu\nu}) \rangle + \gamma_{K, S^\dagger S} |\langle F_S \rangle|^2 \right], \tag{3.5}
\end{aligned}$$

where  $E = i\sqrt{2}g \text{Tr}(Q^\dagger \lambda \psi - \bar{\psi} \lambda \tilde{Q}^\dagger)$ . Note that  $m_{\text{sfermion}}^2$  is of course real, although this is not manifest in (3.5), a consequence of the fact that  $\bar{Q}^2 Q^2(K)$  is not manifestly real.

Finally, for a strongly-coupled theory it is more natural to express the approximations (3.5) in terms of vacuum condensates of IR elementary fields, i.e. the MSSM-restricted mesonic superfield  $\mathcal{M} = \text{Tr}(M)_{(0, N)}$  and the “glueball” superfield  $\mathcal{G} = -(g^2/32\pi^2) \text{tr } W^\alpha W_\alpha$ , leading to

$$\begin{aligned}
M_{\text{gaugino}} &\approx \frac{\pi\alpha_{\text{SM}}}{2N|\xi\langle S \rangle|^2} \left[ 2\xi^* \gamma_{K,K} \langle S^\dagger \mathcal{M}^\dagger \rangle - 2N\gamma_{K,K} \langle \mathcal{G}^\dagger \rangle - \gamma_{K, S^\dagger S} \langle S^\dagger F_S \rangle \right], \\
m_{\text{sfermion}}^2 &\approx \frac{C_2(R)\alpha_{\text{SM}}^2}{4N|\xi\langle S \rangle|^2} \left[ -2\xi^* \gamma_{K,K} \langle S^\dagger F_{\mathcal{M}}^\dagger + F_S^\dagger \mathcal{M}^\dagger \rangle + N\gamma_{K,K} \langle F_{\mathcal{G}} + F_{\mathcal{G}}^\dagger \rangle + \gamma_{K, S^\dagger S} \langle F_S^\dagger F_S \rangle \right]. \tag{3.6}
\end{aligned}$$

Equations (3.4), (3.5) and (3.6) can be easily generalized to more complicated UV theories with several gauge groups and matter fields in different representations. They can also be generalized to closely-related types of mediation like general gaugino mediation [23]. The approximations (3.6) are especially useful since they give an estimate for the MSSM soft SUSY-breaking masses from the knowledge of the vevs of a few IR elementary fields, taking the anomalous-dimension matrix to be of  $\mathcal{O}(1)$ . Indeed only the vacuum structure of both the messenger and the hidden sector is necessary to approximately determine the MSSM superpartner spectrum. The knowledge of the spectrum of messengers does not directly enter the computation.

Note that these approximations should be valid for strongly-coupled theories as well, although the size of the error introduced by truncating the OPE and assuming that cuts extend to the supersymmetric threshold is difficult to estimate in general. The anomalous-dimension terms cannot be computed at strong coupling, but they are expected to be  $\mathcal{O}(1)$ . It is however possible to argue for the functional dependence of the relevant anomalous dimensions at strong coupling. For example,  $\gamma_{K,K}$  should depend on the electric quark mass and electric strong-coupling scale, and since it must be dimensionless it should be expressible by a series in positive powers of

$|\xi\langle S\rangle/\Lambda_e|$  and  $|\xi\langle F_S\rangle/\Lambda_e^2|$ . For  $|\langle F_S\rangle/\xi\langle S\rangle| \ll 1$ , at lowest order one thus expects<sup>8</sup>

$$\gamma_{K,K} \xrightarrow[\text{coupling}]{\text{strong}} \frac{\tilde{N}_c}{16\pi^2} \left| \frac{\xi\langle S\rangle}{\Lambda_e} \right| \delta_{K,K}, \quad \gamma_{K,S^\dagger S} \xrightarrow[\text{coupling}]{\text{strong}} \frac{N\tilde{N}_c}{16\pi^2} |\xi|^2 \delta_{K,S^\dagger S}, \quad (3.7)$$

where  $\delta_{K,K}$  and  $\delta_{K,S^\dagger S}$  are dimensionless numbers of order one. We introduced in (3.7) one-loop factors as well as factors of  $\tilde{N}_c$  and  $N$  to account for the effective number of degrees of freedom propagating in the loops as suggested by the Seiberg dual (see (3.8)).

Furthermore, although the vevs of the appropriate fields in the vacuum of interest are not always calculable in the strongly-coupled regime, it is often possible to approximate them in terms of the relevant scales of the theory under consideration. Therefore the approximations (3.6), which represent the main results of this paper, as well as their generalizations to more complicated models, should be acceptable up to dimensionless numbers of order one. Finally, when weakly-coupled duals exist, it is possible to assess the issues discussed above and directly check that the approximations (3.6) are indeed reliable up to  $\mathcal{O}(1)$  factors, as will be seen in the next section.

In the event that SUSY is discovered at the LHC and that gauge mediation is the relevant means of SUSY-breaking communication, the approximations (3.6) open a rare window into the messenger and the hidden sector: by experimentally measuring the MSSM superpartner spectrum, they allow an approximate determination of some of the vevs of operators in the messenger and the hidden sector. This is reminiscent of QCD sum rules [26] (see also [27] for a nice review and more references), although here the spectrum of hidden-sector resonances is not necessary.

We will now use these equations to investigate the superpartner spectra of sSQCD and its different limits, starting from the computationally-reachable weakly-coupled regime and ending with the often incalculable strongly-coupled regime. To this end, we will use Seiberg duality [10], which for  $SU(N_c)$  sSQCD in the free magnetic phase leads to the following  $SU(\tilde{N}_c \equiv N_f - N_c)$  weakly-coupled dual theory for the matter fields (here the meson  $M$ , the magnetic quarks  $q$  and  $\tilde{q}$ , and the singlet  $S$ ),

$$\begin{aligned} K_m &= \frac{1}{\alpha|\Lambda_e|^2} \text{Tr}(M^\dagger M)_{(0,N_f)} + \frac{1}{\beta} \text{Tr}(q^\dagger q + \tilde{q}\tilde{q}^\dagger)_{(N_f-N_c,N_f)} + S^\dagger S + \dots, \\ W_m &= \frac{1}{\Lambda_d} \text{Tr}(qM\tilde{q})_{(N_f-N_c,N_f)} + \xi S \text{Tr}(M)_{(0,N_f)}, \\ (-1)^{N_f-N_c} \Lambda_d^{N_f} &= \Lambda_e^{3N_c-N_f} \Lambda_m^{3(N_f-N_c)-N_f}. \end{aligned} \quad (3.8)$$

Note that  $\alpha$  and  $\beta$  are positive real dimensionless numbers of order one, and  $\Lambda_e$ ,  $\Lambda_m$  and  $\Lambda_d$  are

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<sup>8</sup>Note that the form of the anomalous current  $K$  is known in terms of magnetic variables around the free supersymmetric and R-symmetric IR CFT in massless SQCD as described by Seiberg duality [24] (see also [25]). However the anomalous dimension computed from this perspective does not lead to the appropriate functional dependence as argued here since we are interested in the ISS SUSY-breaking vacuum.

the electric strong-coupling scale, the magnetic scale and the duality scale respectively.<sup>9</sup> Seiberg duality will allow the determination of the vevs of the relevant IR elementary fields in terms of a few unknowns, therefore providing a direct check of the approximations (3.6).

### 3.1. *sSQCD in the $g \rightarrow 0$ limit: MGM*

In the limit of vanishing hidden-sector gauge coupling, sSQCD is equivalent to MGM with  $N_c$  messenger flavors. In this limit the phenomenology of sSQCD is already well-known, and is easily reproduced with our methods. Indeed, the only non-vanishing vacuum condensate occurs for the MGM singlet  $S$  and the theory is effectively equivalent to MGM with  $N_c$  flavors of messengers as expected. The approximations (3.6) along with the one-loop anomalous-dimension matrix (3.3) thus give (here  $x_S = |\langle F_S \rangle / \xi \langle S \rangle^2|$ )

$$M_{\text{gaugino}} \approx -\frac{\alpha_{\text{SM}}}{4\pi} \frac{\langle F_S \rangle}{\langle S \rangle} \times N_c \times \left\{ g_{\text{approx}}(x_S) = \frac{1}{2} \right\},$$

$$m_{\text{sfermion}}^2 \approx 2 \left( \frac{\alpha_{\text{SM}}}{4\pi} \right)^2 \left| \frac{\langle F_S \rangle}{\langle S \rangle} \right|^2 \times C_2(R) \times N_c \times \left\{ f_{\text{approx}}(x_S) = \frac{1}{2} \right\},$$

which, as already mentioned, are only a factor of 2 smaller than the usually quoted one- and two-loop answers in the limit where  $x_S = 0$  [19],

$$g(x_S) = \frac{1+x_S}{x_S^2} \ln(1+x_S) + \{x_S \rightarrow -x_S\} = 1 + \frac{x_S^2}{6} + \dots,$$

$$f(x_S) = \frac{1+x_S}{x_S^2} \left[ \ln(1+x_S) - 2 \text{Li}_2\left(\frac{x_S}{1+x_S}\right) + \frac{1}{2} \text{Li}_2\left(\frac{2x_S}{1+x_S}\right) \right] + \{x_S \rightarrow -x_S\} = 1 + \frac{x_S^2}{36} + \dots,$$

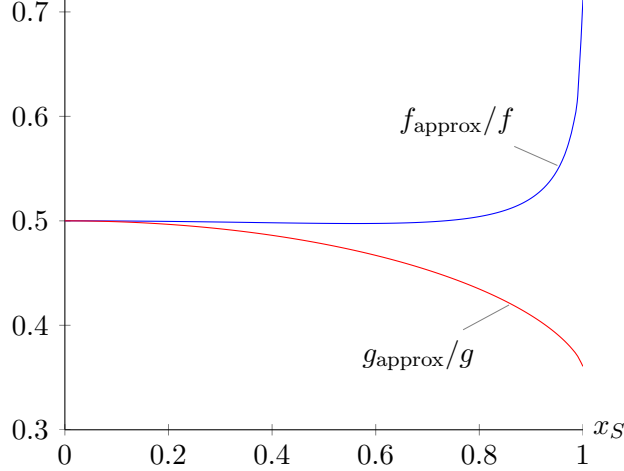
where  $\text{Li}_2(x) = -\int_0^1 dt \frac{\ln(1-xt)}{t}$  is the dilogarithm or Spence function. Note that since the OPE is truncated at lowest order in the SUSY-breaking expansion, it is naturally expected that the approximations (3.6) only capture (part of) the  $x_S = 0$  limit of  $g(x_S)$  and  $f(x_S)$ .

The functions  $g(x_S)$  and  $f(x_S)$ , which are only defined in the region  $0 \leq x_S \leq 1$  in order to avoid tachyonic messengers, do not deviate much from unity, and so the agreement of the OPE with the full answer at one loop for the gauginos and at two loops for the sfermions is reasonable, as can be seen in Fig. 1.

A complete OPE analysis of MGM shows that the method described in [6] and extended here works in the weakly-coupled regime, providing a useful consistency check. Note that it is not easy to use our method to obtain exact results in the weakly-coupled regime. Nevertheless, the simple approximations (3.6) match weakly-coupled computations up to dimensionless numbers of order one, a property which should translate to the strongly-coupled regime as well.

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<sup>9</sup>Due to the freedom in defining the magnetic quarks,  $\beta$ ,  $\Lambda_m$  and  $\Lambda_d$  are not fully determined by the electric theory.



**Fig. 1:**  $g_{\text{approx}}/g$  and  $f_{\text{approx}}/f$  as functions of  $x_S$  for MGM.

### 3.2. $sSQCD$ in the $\langle S \rangle \rightarrow m/\xi$ and $\langle F_S \rangle \rightarrow 0$ limit: $mSQCD$

In the limit where the MGM singlet  $S$  is assumed frozen without an F-term,  $sSQCD$  is nothing else than  $mSQCD$ . The theory is most interesting in the free magnetic phase, given by  $N_c + 1 \leq N_f < 3N_c/2$ , where both a SUSY-preserving phase and a (metastable) SUSY-breaking phase exist [9].

#### 3.2.1. Around the SUSY vacuum

In  $mSQCD$ , although  $\langle \mathcal{M} \rangle$  and  $\langle \mathcal{G} \rangle$  do not vanish at the supersymmetric vacuum, the soft SUSY-breaking masses vanish, as expected, due to the Konishi anomaly [22]. Indeed, although

$$\frac{g^2}{32\pi^2} \langle \text{tr } \lambda \lambda \rangle = [\Lambda e^{3N_c - N_f} \det(\xi \langle S \rangle)]^{\frac{1}{N_c}} e^{2\pi i k / N_c}, \quad (3.9)$$

where (3.9) is valid for any  $N_c$  and  $N_f$  [21], the vacuum condensate for the mesonic superfield is

$$\langle \text{Tr}(\tilde{Q}_i Q^i)_{(N_c, 0)} \rangle = [\Lambda e^{3N_c - N_f} \det(\xi \langle S \rangle)]^{\frac{1}{N_c}} [(\xi \langle S \rangle)^{-1}]_i^i e^{2\pi i k / N_c}, \quad (3.10)$$

as enforced by the Konishi anomaly [22],

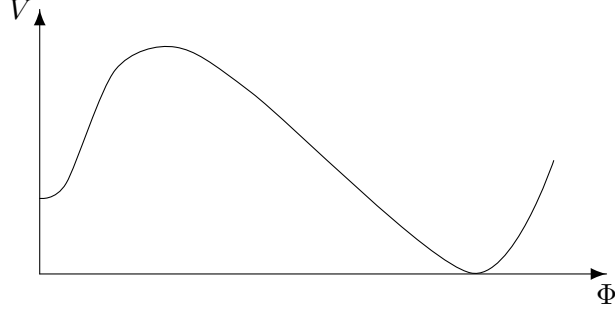
$$\frac{-i}{2\sqrt{2}} \{ \bar{Q}_\alpha, \text{Tr}(\bar{\psi}_i^\alpha Q^i)_{(N_c, 0)} \} = -\xi \langle S \rangle \text{Tr}(\tilde{Q}_i Q^i)_{(N_c, 0)} + \delta_i^i \frac{g^2}{32\pi^2} \text{tr } \lambda \lambda, \quad (3.11)$$

in supersymmetric vacua.<sup>10</sup> In terms of the IR fields this implies that  $\xi \langle S \rangle \langle \mathcal{M} \rangle = N \langle \mathcal{G} \rangle$ . Since all remaining vacuum condensates vanish, the approximations (3.6) lead to a superpartner spectrum consistent with SUSY.

<sup>10</sup>Here the index  $k$  labels the degenerate SUSY vacua which arise from the spontaneous breaking of the discrete global symmetry  $\mathbb{Z}_{2N_c}$  to  $\mathbb{Z}_2$ .

### 3.2.2. Around the ISS vacuum

As shown by ISS [9], mSQCD with small masses has a metastable SUSY-breaking minimum close to the origin of field space. A sketch of the potential of mSQCD is shown in Fig. 2.



**Fig. 2:** A sketch of the potential of mSQCD.

Since the SUSY-breaking scale and the messenger scale are the same in ISS, there is no dimensionless SUSY-breaking parameter to keep track of the order at which SUSY-breaking effects appear in any computation. Thus, in order to compare (3.6) with weakly-coupled computations of the sfermion masses from the dual theory (see Appendix A), it is convenient to distinguish between the SUSY-breaking scale and the messenger scale by introducing two  $\xi$ 's,  $(\xi, \xi_L)$  with  $x_{\mathcal{M}} = \xi_L/\xi$  and  $0 \leq |x_{\mathcal{M}}| \leq 1$ . This effectively splits the mass matrix in two sectors and allows us to keep track of the SUSY-breaking effects.

The location of the SUSY-breaking minimum can be found using the dual theory (3.8) and, in terms of the IR elementary fields (embedding the MSSM into the  $X$ -sector of (3.13)), is given by

$$\langle \mathcal{M} \rangle = \langle \mathcal{G} \rangle = \langle F_{\mathcal{G}} \rangle = 0, \quad \langle F_{\mathcal{M}} \rangle = -N_c \alpha \xi_L^* \langle S^\dagger \rangle |\Lambda_e|^2. \quad (3.12)$$

The ISS vacuum faces an immediate problem for phenomenological applications: it has an accidental R-symmetry and thus constrains to zero Majorana gaugino masses. This can be seen directly from the approximations (3.6) and the vevs (3.12). The sfermion masses, on the other hand, are not constrained by the accidental R-symmetry and are indeed non-zero, as is also clear from (3.6) and the vevs (3.12).

Fixing  $\Lambda_d = \Lambda_m = (-1)^{(N_c - N_f)/(3N_c - N_f)} \Lambda_e$  and using the anomalous dimensions (3.7) the approximated sfermion masses obtained from (3.6) are

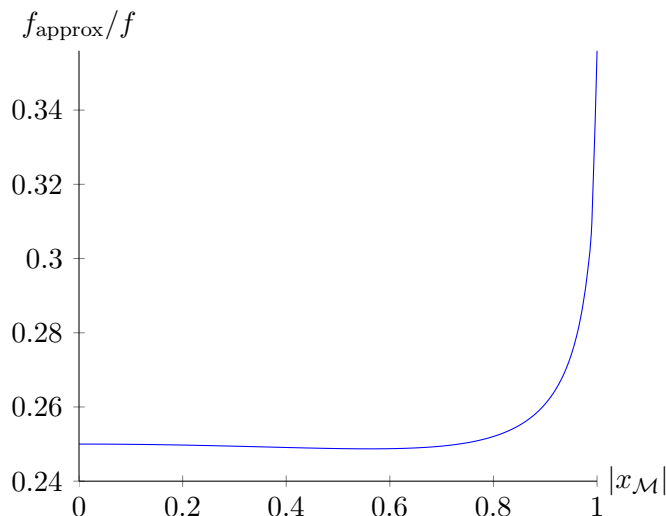
$$m_{\text{sfermion}}^2 \approx 2 \left( \frac{\alpha_{\text{SM}}}{4\pi} \right)^2 |x_{\mathcal{M}}|^2 \alpha \beta |\xi \langle S \rangle \Lambda_e| \times C_2(R) \times \tilde{N}_c \times \left\{ f_{\text{approx}}(x_{\mathcal{M}}) = \frac{4\pi^2}{\tilde{N}_c \beta} \left| \frac{\Lambda_e}{\xi \langle S \rangle} \right| \gamma_{K,K} = \frac{\delta_{K,K}}{4\beta} \right\},$$

while using the dual theory the weakly-coupled computation gives

$$f(x_{\mathcal{M}}) = \frac{1 + |x_{\mathcal{M}}|}{|x_{\mathcal{M}}|^2} \left[ \ln(1 + |x_{\mathcal{M}}|) - 2 \operatorname{Li}_2 \left( \frac{|x_{\mathcal{M}}|}{1 + |x_{\mathcal{M}}|} \right) + \frac{1}{2} \operatorname{Li}_2 \left( \frac{2|x_{\mathcal{M}}|}{1 + |x_{\mathcal{M}}|} \right) \right] + \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\}$$

$$= 1 + \frac{|x_{\mathcal{M}}|^2}{36} + \dots$$

Although  $x_{\mathcal{M}} = 1$  in ISS, our approximations only rely on the lowest-order operators appearing in the OPE and should only capture (part of) the  $x_{\mathcal{M}} = 0$  contributions to  $f(x_{\mathcal{M}})$ , up to a number of order one (as in the MGM case of section 3.1). This is exactly what happens here. Moreover, since the function  $f(x_{\mathcal{M}})$  stays close to unity for all  $x_{\mathcal{M}}$ , the approximations (3.6) are reasonable for  $0 \leq |x_{\mathcal{M}}| \leq 1$  as shown in Fig. 3. Therefore the method developed here gives sensible results



**Fig. 3:**  $f_{\text{approx}}/f$  as function of  $|x_{\mathcal{M}}|$  for mSQCD with  $\beta = \delta_{K,K} = 1$ . Both  $g_{\text{approx}}(x_{\mathcal{M}})$  and  $g(x_{\mathcal{M}})$  vanish and so the corresponding ratio is not plotted here.

even in strongly-coupled theories including higher-order SUSY-breaking corrections.

It is interesting to notice that a full knowledge of the OPE could possibly lead to a computation of the anomalous dimensions of relevant operators in mSQCD following the method described here, as was done for MGM in [6]. For more details the reader is referred to section 3.3.2 and [9].

### 3.3. sSQCD in the free magnetic phase

Here we explore sSQCD for  $N_c + 1 \leq N_f < 3N_c/2$ . As mentioned above, in mSQCD dynamical SUSY breaking in metastable vacua occurs for this range of  $N_f$  close to the origin of field space [9]. Although the electric theory is strongly coupled, Seiberg duality allows one to establish the presence of SUSY breaking. In this subsection we also use Seiberg duality to understand SUSY breaking in sSQCD close to the origin of field space.



### 3.3.1. Around the would-be SUSY vacuum

Let us first discuss the fate of the would-be SUSY vacuum of mSQCD in the full sSQCD theory. For  $|\langle F_S \rangle / \xi \langle S \rangle^2| \ll 1$  one would expect that the vevs of the glueball and mesonic superfields are only slightly perturbed compared to their mSQCD values (3.9) and (3.10). Moreover, from the point of view of the sSQCD fields, SUSY is explicitly broken. One should thus expect that the SUSY vacuum of mSQCD becomes a SUSY-breaking vacuum in sSQCD. Since small instantons are relevant, it is impossible to compute the vevs of the glueball and mesonic fields from instanton techniques without a full knowledge of the hidden-sector theory. It is nevertheless possible to estimate the vev of the lowest component of the mesonic superfield from the superpotential and the Kähler potential (3.8), leading to

$$\langle \mathcal{M} \rangle = [\xi^{N_f - N_c} \langle S \rangle^{N_f - N_c} \Lambda_e^{3N_c - N_f}]^{\frac{1}{N_c}} \left[ 1 + \frac{N_c - N_f}{N_c} \frac{1}{\alpha |\xi|^2} \left( \frac{\xi^* \langle S^\dagger \rangle}{\Lambda_e^*} \right)^{\frac{N_f}{N_c}} \frac{\langle F_S^\dagger \rangle \Lambda_e^{*2}}{\langle S^\dagger \rangle^2 \langle S \rangle \Lambda_e} + \dots \right].$$

One could then use the Konishi anomaly (3.11) to obtain the vev of the glueball superfield, but since the vacuum is expected to be non-supersymmetric, this approach is inconclusive. A complete knowledge of the hidden sector seems thus necessary to determine the characteristics of the superpartner spectrum around this vacuum.

### 3.3.2. Around the ISS-like vacuum

Around the origin of field space it is more convenient to use the dual theory as given by (3.8), but with canonically-normalized matter fields  $\Phi$ ,  $\varphi$  and  $\tilde{\varphi}$ . The superpotential becomes

$$W_m = h \text{Tr } \varphi \Phi \tilde{\varphi} - h \Psi \text{Tr } \Phi,$$

where  $\Psi$  is a background field with  $\langle \Psi \rangle = \mu^2 + \theta^2 \mu_F^3$ . The parameter  $\mu_F$  is the source of R-symmetry breaking in our example. With the parametrization

$$\Phi = \begin{pmatrix} Y_{\tilde{N}_c \times \tilde{N}_c} & Z_{\tilde{N}_c \times N_c}^T \\ \tilde{Z}_{N_c \times \tilde{N}_c} & X_{N_c \times N_c} \end{pmatrix}, \quad \varphi^T = \begin{pmatrix} \chi_{\tilde{N}_c \times \tilde{N}_c} \\ \rho_{N_c \times \tilde{N}_c} \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \tilde{\chi}_{\tilde{N}_c \times \tilde{N}_c} \\ \tilde{\rho}_{N_c \times \tilde{N}_c} \end{pmatrix}, \quad (3.13)$$

the scalar potential becomes

$$\begin{aligned} V = & N_f |h \mu^2|^2 + h \mu_F^3 \text{Tr}(Y + X) + h^* \mu_F^{*3} \text{Tr}(Y^\dagger + X^\dagger) \\ & + |h|^2 \text{Tr}[-\mu^2(\tilde{\chi}^\dagger \chi^* + \tilde{\rho}^\dagger \rho^*) - \mu^{*2}(\chi^T \tilde{\chi} + \rho^T \tilde{\rho}) \\ & + \tilde{\chi}^\dagger(Y^\dagger Y + \tilde{Z}^\dagger \tilde{Z})\tilde{\chi} + \tilde{\rho}^\dagger(Z^* Z^T + X^\dagger X)\tilde{\rho} + \tilde{\rho}^\dagger(Z^* Y + X^\dagger \tilde{Z})\tilde{\chi} \\ & + \tilde{\chi}^\dagger(Y^\dagger Z^T + \tilde{Z}^\dagger X)\tilde{\rho} + \chi^\dagger(Y^* Y^T + Z^\dagger Z)\chi + \rho^\dagger(\tilde{Z}^* \tilde{Z}^T + X^* X^T)\rho \\ & + \rho^\dagger(\tilde{Z}^* Y^T + X^* Z)\chi + \chi^\dagger(Y^* \tilde{Z}^T + Z^\dagger X^T)\rho + (\chi^T \chi^* + \rho^T \rho^*)(\tilde{\chi}^\dagger \tilde{\chi} + \tilde{\rho}^\dagger \tilde{\rho})]. \end{aligned}$$

As in the ISS case, the rank condition implies that SUSY is broken with  $F_X^\dagger = h\mu^2$ , and a minimum should develop around the origin of field space, which can be conveniently described with the following ansatz:

$$\langle \Phi \rangle = \begin{pmatrix} Y_0 & 0 \\ 0 & X_0 \end{pmatrix}, \quad \langle \varphi^T \rangle = \begin{pmatrix} q_0 \\ 0 \end{pmatrix}, \quad \langle \tilde{\varphi} \rangle = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix}.$$

Assuming  $\tilde{q}_0 = q_0 = q$  the scalar potential is minimized in (almost) all directions when

$$Y_0 = -\frac{\mu_F^{*3}}{h(|q_0|^2 + |\tilde{q}_0|^2)} = -\frac{\mu_F^{*3}}{2h|q|^2}, \quad (3.14)$$

$$q = \frac{1}{3}\mu(1 + H^{1/3} + H^{-1/3})^{1/2}, \quad \text{where} \quad H = 1 - \frac{27}{2}|\epsilon| \left( |\epsilon| - \sqrt{|\epsilon|^2 - \frac{4}{27}} \right).$$

Here  $\epsilon = \mu_F^3/2h^*\mu^{*2}\mu$  and it is assumed small. The constraint on  $q$  comes from minimization in the  $\tilde{\chi}$ -direction leading to the condition

$$|q|^6 - \mu^{*2}q^2|q|^2 + \left| \frac{\mu_F^3}{2h} \right|^2 = 0,$$

which requires that  $q/\mu \in \mathbb{R}$ . Keeping the solution<sup>11</sup> for which  $q \xrightarrow[\mu_F \rightarrow 0]{} \mu$  leads to the vev mentioned above. For a well-defined  $q$  one needs  $|\epsilon| \leq \frac{2\sqrt{3}}{9}$  which is easily satisfied for small  $|\epsilon|$ . For small  $\mu_F$  (or  $\epsilon$ ), (3.14) can be approximated by

$$Y_0 = -\frac{\mu_F^{*3}}{2h|\mu|^2} + \dots, \quad q = \mu \left( 1 - \frac{1}{2}|\epsilon|^2 + \dots \right).$$

The scalar potential is stabilized in all but the  $X$ -direction. As opposed to the ISS case where  $X$  is a flat direction of  $V$ , here  $X$  is a runaway direction at tree level and  $V$  slopes down in the  $X$ -direction. Since the runaway behavior is dictated by the small deformation  $\mu_F$ , it is expected that the one-loop Coleman–Weinberg potential stabilizes the runaway direction close to the origin of field space, thus leading to spontaneous breaking of the accidental R-symmetry of the ISS model and allowing for non-vanishing gaugino masses.

To calculate the Coleman–Weinberg potential [28] for a general supersymmetric theory with  $n$  chiral superfields  $\Phi^i$ , canonical Kähler potential, and superpotential  $W(\Phi)$ , we need the mass matrices for scalar and spin- $\frac{1}{2}$  fields, given respectively by the  $2n \times 2n$  matrices

$$\mathbb{M}_0^2 = \begin{pmatrix} W^{\dagger ik} W_{kj} & W^{\dagger ijk} W_k \\ W_{ijk} W^{\dagger k} & W_{ik} W^{\dagger kj} \end{pmatrix} \quad \text{and} \quad \mathbb{M}_{1/2}^2 = \begin{pmatrix} W^{\dagger ik} W_{kj} & 0 \\ 0 & W_{ik} W^{\dagger kj} \end{pmatrix},$$

with  $W_i \equiv \partial W / \partial \Phi^i$  and similarly for the rest, where the derivatives are to be evaluated at the vevs computed for the zero components of the chiral superfields.

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<sup>11</sup>The other solutions lead to tachyons.

In the case of supersymmetric theories, where quadratic divergences cancel among bosons and fermions,

$$\text{STr } \mathbb{M}^2 \equiv \text{Tr } \mathbb{M}_0^2 - \text{Tr } \mathbb{M}_{1/2}^2 = 0, \quad (3.15)$$

the Coleman–Weinberg potential takes the form

$$V_{\text{CW}} = \frac{1}{64\pi^2} \text{STr } \mathbb{M}^4 \ln \frac{\mathbb{M}^2}{\Lambda^2} \equiv \frac{1}{64\pi^2} \left[ \text{Tr } \mathbb{M}_0^4 \left( \ln \frac{\mathbb{M}_0^2}{4\Lambda^2} + \frac{1}{2} \right) - \text{Tr } \mathbb{M}_{1/2}^4 \left( \ln \frac{\mathbb{M}_{1/2}^2}{4\Lambda^2} + \frac{1}{2} \right) \right],$$

where  $\Lambda$  is the cutoff scale and plays no role in the following. We are therefore interested in  $V_{\text{CW}}$  as a function of the runaway direction  $X$ ,  $V_{\text{CW}}(X)$ . Due to the supertrace relation (3.15), we only have to consider the mass matrices for the  $(\rho, Z)$  sector, since this is the only sector in which the spectrum is non-supersymmetric at tree level.

The mass eigenstates for the messenger sectors are fairly complicated. To simplify the analysis we choose to compute them at order  $\epsilon$ , leading to

$$\begin{aligned} \tilde{m}_1^2 &= |h\mu|^2 \frac{\epsilon x + \epsilon^* x^*}{1 + |x|^2}, \\ \tilde{m}_2^2 &= |h\mu|^2 \left( 1 + |x|^2 - \frac{\epsilon x + \epsilon^* x^*}{1 + |x|^2} \right), \\ \tilde{m}_3^2 &= |h\mu|^2 \left( \frac{3}{2} + \frac{1}{2}|x|^2 - \frac{1}{2}(1 + 6|x|^2 + |x|^4)^{1/2} \right. \\ &\quad \left. + \frac{1 + |x|^2 - (1 + 6|x|^2 + |x|^4)^{1/2}}{1 + 6|x|^2 + |x|^4 - (1 + |x|^2)(1 + 6|x|^2 + |x|^4)^{1/2}} (\epsilon x + \epsilon^* x^*) \right), \\ \tilde{m}_4^2 &= |h\mu|^2 \left( \frac{3}{2} + \frac{1}{2}|x|^2 + \frac{1}{2}(1 + 6|x|^2 + |x|^4)^{1/2} \right. \\ &\quad \left. + \frac{1 + |x|^2 + (1 + 6|x|^2 + |x|^4)^{1/2}}{1 + 6|x|^2 + |x|^4 + (1 + |x|^2)(1 + 6|x|^2 + |x|^4)^{1/2}} (\epsilon x + \epsilon^* x^*) \right), \end{aligned} \quad (3.16)$$

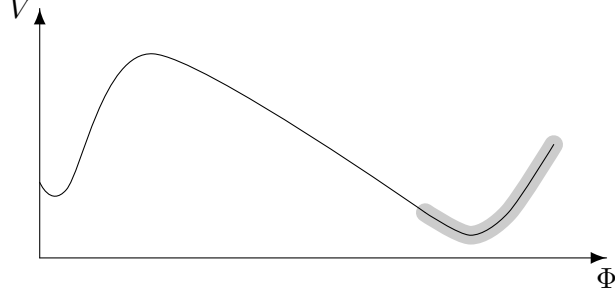
for the bosonic mass eigenstates and

$$\begin{aligned} m_1^2 &= |h\mu|^2 \left( 1 + \frac{1}{2}|x|^2 - \frac{1}{2}|x|(4 + |x|^2)^{1/2} \right), \\ m_2^2 &= |h\mu|^2 \left( 1 + \frac{1}{2}|x|^2 + \frac{1}{2}|x|(4 + |x|^2)^{1/2} \right), \end{aligned} \quad (3.17)$$

for the fermionic mass eigenstates. Note that to simplify the notation we introduced  $x = X/\mu$ . Moreover, it is important to notice that  $\tilde{m}_1$  vanishes exactly once higher-order corrections are introduced since it corresponds to a Goldstone boson.

Including the Coleman–Weinberg potential with corrections up to  $\mathcal{O}(\epsilon)$  terms, the runaway in the  $X$ -direction is found to be stabilized at

$$X_0 = -\frac{16\pi^2 + \tilde{N}_c |h|^2 \ln 2}{\tilde{N}_c |h|^2 (\ln 4 - 1)} \epsilon^* \mu,$$



**Fig. 4:** A sketch of the potential of sSQCD. The shading indicates that our analysis of the spectrum in the corresponding region, i.e. around and past the would-be SUSY vacuum, is not conclusive.

and a minimum appears close to the origin in field space. As we have explained, SUSY is also broken in the faraway vacuum. A sketch of the potential can be seen in Fig. 4.

To make use of (3.6) we relate the canonically-normalized IR fields to the UV elementary fields with the help of the following dictionary:

$$\begin{aligned} \varphi &= \frac{q}{\sqrt{\beta}}, & \tilde{\varphi} &= \frac{\tilde{q}}{\sqrt{\beta}}, & \Phi &= \frac{M}{\sqrt{\alpha}\Lambda_e}, \\ h &= \frac{\sqrt{\alpha}\beta\Lambda_e}{\Lambda_d}, & \mu^2 &= -\frac{\xi\langle S\rangle\Lambda_d}{\beta}, & \mu_F^3 &= -\frac{\xi\langle F_S\rangle\Lambda_d}{\beta}. \end{aligned}$$

As already mentioned, one can choose to fix  $\Lambda_d = \Lambda_m = (-1)^{(N_c - N_f)/(3N_c - N_f)}\Lambda_e$  and describe the results in terms of  $\alpha$  and  $\beta$ , which leads to ( $N = N_c$ )

$$\langle \mathcal{M} \rangle = \frac{N_c \sigma(x_{\mathcal{M}})}{2\beta} \left| \frac{\Lambda_e}{\xi\langle S \rangle} \right| \xi^* \langle F_S^\dagger \rangle, \quad \langle F_{\mathcal{M}} \rangle = -N_c \alpha \xi_L^* \langle S^\dagger \rangle |\Lambda_e|^2,$$

when embedding the MSSM gauge group into the  $X$ -sector of (3.13). Here  $\xi_L$  has been introduced, as in the ISS case, to keep track of the SUSY-breaking effects, and  $\sigma(x_{\mathcal{M}})$  encodes the position of the minimum as a function of the SUSY-breaking effects,

$$\begin{aligned} \sigma(x_{\mathcal{M}}) &= \frac{16\pi^2 + \tilde{N}_c \alpha \beta^2 a}{\tilde{N}_c \alpha \beta^2 b} x_{\mathcal{M}}^*, \\ a &= \frac{1}{2x_{\mathcal{M}}} [(1 + |x_{\mathcal{M}}|) \ln(1 + |x_{\mathcal{M}}|) + \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\}] \xrightarrow{x_{\mathcal{M}} \rightarrow 1} \ln 2, \\ b &= \frac{1}{2|x_{\mathcal{M}}|} [(1 + |x_{\mathcal{M}}|)^2 \ln(1 + |x_{\mathcal{M}}|) - |x_{\mathcal{M}}| - \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\}] \xrightarrow{x_{\mathcal{M}} \rightarrow 1} \ln 4 - 1. \end{aligned}$$

Using the anomalous dimensions (3.7), the superpartner spectrum at order  $\mathcal{O}(\mu_F^3) \sim \mathcal{O}(\langle F_S \rangle)$  is

thus

$$\begin{aligned}
M_{\text{gaugino}} &\approx \frac{\alpha_{\text{SM}}}{4\pi} \frac{\langle F_S \rangle}{\langle S \rangle} \times \tilde{N}_c \times \left\{ g_{\text{approx}}(x_{\mathcal{M}}) = \sigma^*(x_{\mathcal{M}}) x_{\mathcal{M}}^* \frac{2\pi^2}{\tilde{N}_c \beta} \left| \frac{\Lambda_e}{\xi \langle S \rangle} \right| \gamma_{K,K} - \frac{2\pi^2}{N_c \tilde{N}_c |\xi|^2} \gamma_{K,S^\dagger S} = \right. \\
&\quad \left. = \sigma^*(x_{\mathcal{M}}) x_{\mathcal{M}}^* \frac{\delta_{K,K}}{8\beta} - \frac{\delta_{K,S^\dagger S}}{8} \right\}, \\
m_{\text{sfermion}}^2 &\approx 2 \left( \frac{\alpha_{\text{SM}}}{4\pi} \right)^2 |x_{\mathcal{M}}|^2 \alpha \beta |\xi \langle S \rangle \Lambda_e| \times C_2(R) \times \tilde{N}_c \\
&\quad \times \left\{ f_{\text{approx}}(x_{\mathcal{M}}) = \frac{4\pi^2}{\tilde{N}_c \beta} \left| \frac{\Lambda_e}{\xi \langle S \rangle} \right| \gamma_{K,K} = \frac{\delta_{K,K}}{4\beta} \right\}, \tag{3.18}
\end{aligned}$$

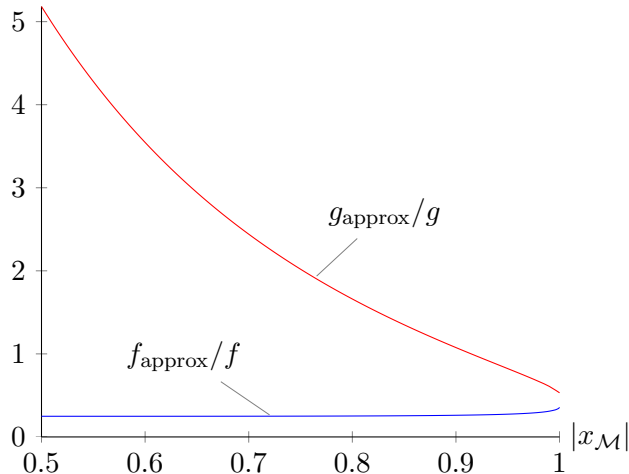
and can be compared to the weakly-coupled computation which gives

$$\begin{aligned}
g(x_{\mathcal{M}}) &= \left[ \frac{1 + |x_{\mathcal{M}}|}{|x_{\mathcal{M}}|^2} \ln(1 + |x_{\mathcal{M}}|) + \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\} \right] \\
&\quad + \frac{\sigma^*(x_{\mathcal{M}})}{2x_{\mathcal{M}}|x_{\mathcal{M}}|} [3|x_{\mathcal{M}}| - (3 + 4|x_{\mathcal{M}}| + |x_{\mathcal{M}}|^2) \ln(1 + |x_{\mathcal{M}}|) - \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\}] \\
&= 1 + \frac{|x_{\mathcal{M}}|^2}{6} + \dots + \sigma^*(x_{\mathcal{M}}) \left( \frac{x_{\mathcal{M}}^* |x_{\mathcal{M}}|^2}{15} + \dots \right), \\
f(x_{\mathcal{M}}) &= \frac{1 + |x_{\mathcal{M}}|}{|x_{\mathcal{M}}|^2} \left[ \ln(1 + |x_{\mathcal{M}}|) - 2 \text{Li}_2 \left( \frac{|x_{\mathcal{M}}|}{1 + |x_{\mathcal{M}}|} \right) + \frac{1}{2} \text{Li}_2 \left( \frac{2|x_{\mathcal{M}}|}{1 + |x_{\mathcal{M}}|} \right) \right] + \{|x_{\mathcal{M}}| \rightarrow -|x_{\mathcal{M}}|\} \\
&= 1 + \frac{|x_{\mathcal{M}}|^2}{36} + \dots. \tag{3.19}
\end{aligned}$$

At order  $\mathcal{O}(\langle F_S \rangle)$  the sSQCD sfermion masses are the same as the mSQCD sfermion masses. Note that the functional dependence of the anomalous dimension  $\gamma_{K,K}$ , necessary for the approximate gaugino masses to match the weakly-coupled computation, is the same as the one expected from the sfermion masses. This strongly suggests that the functional dependence of  $\gamma_{K,K}$  is indeed proportional to  $|\xi \langle S \rangle / \Lambda_e|$ .

As for the mSQCD case,  $x_{\mathcal{M}} = 1$  but by truncating the OPE the results (3.18) should only capture the lowest-order contribution in the  $x_{\mathcal{M}}$ -expansion of  $g(x_{\mathcal{M}})$  and  $f(x_{\mathcal{M}})$  up to  $\mathcal{O}(1)$  factors, as can be seen directly. Note however that the power in  $|x_{\mathcal{M}}|$  of the spontaneous R-symmetry breaking contribution to the gaugino mass, denoted by  $\sigma(x_{\mathcal{M}})$ , does not exactly match the weakly-coupled computation: it is off by a factor of  $|x_{\mathcal{M}}|^2$ . This suggests that all OPE contributions of the same type must be included to appreciate the suppression seen at small dynamical SUSY breaking, i.e. for small  $|x_{\mathcal{M}}|$ . Yet, this point is of no relevance since the metastable SUSY-breaking minimum disappears for small  $|x_{\mathcal{M}}|$ , indeed  $\langle \mathcal{M} \rangle \xrightarrow{x_{\mathcal{M}} \rightarrow 0} \infty$ . This is clear since for fixed  $\epsilon$ , the Coleman–Weinberg potential cannot compete against the runaway when  $|x_{\mathcal{M}}|$  is too small. The value of  $x_{\mathcal{M}}$  at which the minimum disappears can be estimated from

the constraint that the messenger masses must be all non-tachyonic. Using the messenger masses at order  $\epsilon$ , this constraint is obtained from the fermionic messenger mass eigenstates (3.17). In Fig. 5 we plot our results for  $0.5 \leq |x_{\mathcal{M}}| \leq 1$ .



**Fig. 5:**  $g_{\text{approx}}/g$  and  $f_{\text{approx}}/f$  as functions of  $|x_{\mathcal{M}}|$  for sSQCD with  $\beta = \delta_{K,K} = \delta_{K,S^\dagger S} = 1$  and  $\tilde{N}_c = 2$ .

Note that the gaugino approximation overestimates the mass if all dimensionless numbers are positive.<sup>12</sup> Overall, the method described here gives sensible results even for strongly-coupled theories of SUSY-breaking. Again, a complete knowledge of the OPE could allow a determination of the anomalous dimensions of relevant operators of sSQCD using these methods.

Finally, even though it is not the main purpose of this paper, it is of interest to discuss some of the phenomenology of this new deformation. From the IR point of view, sSQCD is reminiscent of the multitrace deformation discussed in [12]. The main difference can be found in the fermionic sector, where the goldstino also has a component in the  $\psi_S$  direction. As such, multitrace deformations are not needed here to give reasonable masses to the fermionic components of  $X$ . The phenomenology of sSQCD is thus very similar to the phenomenology of [12].

At this point one may observe that there appears to be a contradiction between our result (3.18) for  $M_{\text{gaugino}}$ , using also the explicitly computed  $g(x_{\mathcal{M}})$  of (3.19), and the general result of small first-order gaugino mass of Komargodski and Shih [29]. However, this is not so: our example is in a sense modular. The gaugino mass appears proportional to  $\langle F_S \rangle / \langle S \rangle$ , for it arises from the extra SUSY-breaking sector we have included. This can then be thought of as a separate hidden sector, with ISS as the messenger sector. The treatment of Komargodski and Shih does not constrain such models.

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<sup>12</sup>From the sfermion mass (3.18) it is clear that  $\delta_{K,K}/\beta$  is positive and thus  $\delta_{K,K}$  must be positive.

## 4. Discussion and conclusion

In this paper we have used the results of [6] to further illustrate how the OPE can be used to understand superpartner spectra in the MSSM in the context of gauge mediation. Although delivering only approximate answers, our methods do capture the essential physics of soft-mass generation in the MSSM. This becomes possible through the UV-IR splitting achieved by the OPE. The methods developed here lead to approximations valid up to order one numbers both at weak and strong coupling, as can be checked explicitly for strongly-coupled theories with weakly-coupled duals. For strongly-coupled theories of SUSY breaking without weakly-coupled duals, the logic can be inverted and the approximations discussed here might allow us to argue for the functional dependence of relevant anomalous dimensions, which are in practice technically very difficult to calculate.

Using similar techniques one should also be able to perform approximate computations of total cross-sections from the visible sector to the hidden sector, which could be very useful in the event that SUSY is discovered at the LHC.

Our methods were applied here to a new deformation of SQCD, where an additional spontaneous breaking of SUSY is considered. This arises from the F-term vev of a spurion  $S$ , whose zero component supplies the quark masses in SQCD. This deformation moves the ISS vacuum away from the origin and thus induces a breaking of the accidental R-symmetry. Consequently, Majorana gaugino masses are allowed in this ISS-like vacuum. Note that there are no SUSY vacua with our deformation of SQCD. An obvious extension of our work would be to study the  $\mu/B_\mu$  problem in strongly-coupled models, although this is bound to be more model-dependent.

In (3.6) and (3.18), the main results of this paper, the soft masses are parametrized by entries of the anomalous-dimension matrix  $\gamma$  between the current  $K$  and the spurionic operator  $S^\dagger S$ . The calculation of  $\gamma$  can be easily done in the UV, where the electric theory is under control, with the one-loop result (3.3). One could then imagine using magnetic variables to express  $\gamma$  in a form useful in the IR, but the presence of the electric coupling  $g$  in (3.3) complicates matters. A direct calculation of  $\gamma$  in the IR of SQCD, around the SUSY-breaking minimum, using the magnetic description from the outset, is thus more desirable. However the meaning of the current  $K$  in terms of the magnetic dual fields is not clear *a priori*. It would be interesting to carry out in mSQCD the computations done for MGM, and then determine some of the relevant anomalous dimensions of mSQCD operators.

Finally, the approximation (3.6) for the gaugino masses in theories with only dynamical SUSY breaking does not depend on any F-terms. Corrections from the small explicit R-symmetry breaking, necessary to generate non-vanishing Majorana gaugino masses, modify the approximation (3.6), however the main contribution resides in the vev of the MSSM-restricted mesonic superfield, which carries the appropriate R-charge. For hidden-sector gauge groups that are completely Higgsed, this implies that the vev of the MSSM-restricted mesonic superfield must either vanish

or blow up as the dynamical SUSY-breaking effect is taken to zero. This observation suggests that to obtain acceptable phenomenology, with  $|M_{\text{gaugino}}/m_{\text{sfermion}}|$  of order one, the spontaneous R-symmetry breaking must be non-negligible.

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### Appendix A. Superpartner spectra in weakly-coupled theories

Superpartner spectra can be computed directly from the messenger sector in weakly-coupled theories of SUSY breaking. Although the result is well-known for simple messenger sectors, as for example in MGM [19], for general messenger sectors this is not the case (for a derivation using GGM, see [30]).

Consider a messenger sector consisting of  $n$  chiral superfields  $\Phi_i$  and  $\tilde{\Phi}_i$  transforming in a vector-like representation  $R + \bar{R}$  of the MSSM with arbitrary mass matrices  $(\mathbb{M}_0^2)_{2n \times 2n}$  and  $(\mathbb{M}_{1/2})_{n \times n} = W_{ij}$  such that

$$\mathcal{L} \supset - \begin{pmatrix} \phi_i^* & \tilde{\phi}_i \end{pmatrix} (\mathbb{M}_0^2)_{ij} \begin{pmatrix} \phi_j \\ \tilde{\phi}_j^* \end{pmatrix} - \begin{pmatrix} \tilde{\psi}_i \end{pmatrix} (\mathbb{M}_{1/2})_{ij} \begin{pmatrix} \psi_j \end{pmatrix} - \text{h.c.},$$

where  $(\phi_i, \psi_i)$  are the bosonic and fermionic components of  $\Phi_i$  and similarly for  $\tilde{\Phi}_i$ . Introducing unitary matrices  $U_b$ ,  $U_f$  and  $\tilde{U}_f$  which diagonalize the mass matrices,

$$\tilde{m}_i^2 \delta_{ij} = (U_b \mathbb{M}_0^2 U_b^\dagger)_{ij}, \quad m_i \delta_{ij} = (\tilde{U}_f^* \mathbb{M}_{1/2} U_f^\dagger)_{ij},$$

with  $\tilde{m}_i$  and  $m_i$  the (real positive) bosonic and fermionic mass eigenvalues respectively, the gaugino and sfermion masses are given by

$$M_{\text{gaugino}} = -\frac{\alpha_{\text{SM}}}{\pi} C(R) \mathcal{G}, \quad m_{\text{sfermion}}^2 = \left( \frac{\alpha_{\text{SM}}}{4\pi} \right)^2 C_2(R_{\text{sfermion}}) C(R) \mathcal{F}^2, \quad (\text{A.1})$$

where  $C(R) = \frac{1}{2}$  for the fundamental representation and

$$\mathcal{G} = \sum_{i=1}^{2n} \sum_{j,k,l=1}^n (U_b)_{ik} (U_b^*)_{i,n+l} (U_f^\dagger)_{kj} (\tilde{U}_f^\dagger)_{lj} m_j \left[ \ln \left( \frac{\Lambda^2}{m_j^2} \right) - \frac{\tilde{m}_i^2}{\tilde{m}_i^2 - m_j^2} \ln \left( \frac{\tilde{m}_i^2}{m_j^2} \right) \right],$$



$$\begin{aligned}
\mathcal{F}^2 = & \sum_{i=1}^{2n} \tilde{m}_i^2 \ln(\tilde{m}_i^2) [4 + \ln(\tilde{m}_i^2)] + 4 \sum_{i=1}^n m_i^2 \ln(m_i^2) [-2 + \ln(m_i^2)] \\
& + \sum_{i,j,k,l=1}^{2n} (-1)^{[(k-1)/n] + [(l-1)/n]} (U_b)_{ik} (U_b^\dagger)_{kj} (U_b)_{jl} (U_b^\dagger)_{li} \\
& \times \tilde{m}_i^2 \left[ -\ln(\tilde{m}_j^2) \ln(\tilde{m}_j^2) + 2 \ln(\tilde{m}_i^2) \ln(\tilde{m}_j^2) - 2 \text{Li}_2 \left( 1 - \frac{\tilde{m}_i^2}{\tilde{m}_j^2} \right) \right] \\
& + 2 \sum_{i=1}^{2n} \sum_{j,k,l=1}^n \left[ (U_b^\dagger)_{ki} (U_f)_{jk} (U_b)_{il} (U_f^\dagger)_{lj} + (U_b^\dagger)_{n+k,i} (\tilde{U}_f^*)_{jk} (U_b)_{i,n+l} (\tilde{U}_f^T)_{lj} \right] \\
& \times \left\{ \tilde{m}_i^2 \left[ \ln(m_j^2) \ln(m_j^2) - 2 \ln(\tilde{m}_i^2) \ln(m_j^2) + 2 \text{Li}_2 \left( 1 - \frac{\tilde{m}_i^2}{m_j^2} \right) - 2 \text{Li}_2 \left( 1 - \frac{m_j^2}{\tilde{m}_i^2} \right) \right] \right. \\
& \left. + m_j^2 \left[ \ln(\tilde{m}_i^2) \ln(\tilde{m}_i^2) - 2 \ln(\tilde{m}_i^2) \ln(m_j^2) + 2 \text{Li}_2 \left( 1 - \frac{\tilde{m}_i^2}{m_j^2} \right) + 2 \text{Li}_2 \left( 1 - \frac{m_j^2}{\tilde{m}_i^2} \right) \right] \right\}.
\end{aligned}$$

The diagrams leading to (A.1) can be found in [19]. Note that due to the magic of SUSY, the cutoff  $\Lambda$  does not appear in the gaugino masses. Here  $\text{Li}_2(x) = -\int_0^1 dt \frac{\ln(1-xt)}{t}$  is the dilogarithm or Spence function.

Note that, although the messenger spectrum of MGM, mSQCD and sSQCD are quite different, the superpartner spectra are given in terms of the same functions  $g(x)$  and  $f(x)$  (when the spontaneous R-symmetry breaking contribution is discarded in the sSQCD case).

As a final point, note that it is straightforward to include extra messengers transforming under different representations of the MSSM gauge group.

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